BMES CELL TEAM
SPRING 2021

Aside on First Order Differential Equations

If we are given a differential equation of the form of equation (1), we can use the *integrating factor method*.

$$\frac{dy}{dt} + y \cdot \alpha(t) = \beta(t) \tag{1}$$

The integrating factor is defined as:

$$IF = \exp\left(\int \alpha(t)dt\right)$$

Multiply both sides of the ODE by the integrating factor:

$$\exp\left(\int \alpha(t)dt\right)\left(\frac{dy}{dt} + y \cdot \alpha(t)\right) = \exp\left(\int \alpha(t)dt\right)\beta(t)$$

By the inverse product and chain rule, we can rewrite the left hand side of the equation:

$$\left(\frac{d}{dt}\right)\left(y \cdot \exp\left(\int \alpha(t)dt\right)\right) = \exp\left(\int \alpha(t)dt\right)\beta(t)$$

Now we can integrate both sides and solve for y:

$$\int y \cdot \exp\left(\int \alpha(t)dt\right) d = \int \exp\left(\int \alpha(t)dt\right)\beta(t)dt$$

$$y \cdot \exp\left(\int \alpha(t)dt\right) = \int \exp\left(\int \alpha(t)dt\right)\beta(t)dt + K$$

$$\boxed{y = \frac{1}{\exp\left(\int \alpha(t)dt\right)} \int \exp\left(\int \alpha(t)dt\right)\beta(t)dt + \frac{K}{\exp\left(\int \alpha(t)dt\right)}}$$

Notes:

- While you *could* just apply the boxed formula to your problem, you should still show all your work for maximum partial credit
- Going back to equation (1), if you replace $\beta(t)$ with 0, then your equation would be a *homogenous* equation. This is why this method also works for homogenous equations.