BMES CELL TEAM
SPRING 2021

Aside on Second Order Homogeneous Differential Equations

A second order homogenous ODE is one that can be manipulated into the form of:

$$\frac{d^2 f}{dx^2} + a\frac{df}{dx} + bf = 0$$

Note that the coefficient of the 2nd order term is 1.

To solve this differential equation, we set an arbitrary value equal to the first derivative:

$$\gamma \equiv \frac{d}{dx}$$

Now, we can write the differential equation as a basic quadratic equation:

$$\gamma^2 + a\gamma + b = 0$$

Now, apply the quadratic formula to solve for the roots:

$$\gamma = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

Use the table below to find the solution:

Roots (γ_1, γ_2)	**Solution**
If γ_1 and γ_2 are real and distinct	$f(x) = C_1 \exp(\gamma_1 x) + C_2 \exp(\gamma_2 x)$
If γ_1 and γ_2 are real and equal $(\gamma_1 = \gamma_2 = \gamma_0)$	$f(x) = C_1 \exp(\gamma_0 x) + C_2 x \exp(\gamma_0 x)$
If γ_1 and γ_2 are complex $(\gamma_1 = \mu + i\nu)$ and $(\gamma_2 = \mu - i\nu)$	$f(x) = \exp(\mu x)[C_1 \cos(\nu x) + C_2 \sin(\nu x)]$
If γ_1 and γ_2 are imaginary $(\gamma_1 = +i\nu)$ and $(\gamma_2 = -i\nu)$	$f(x) = C_1 \cos(\nu x) + C_2 \sin(\nu x)$

Special Case: If γ_1 and γ_2 are real and opposite $(+/-)$, we can use either of the following solutions:

- $f(x) = C_1 \exp(\gamma_1 x) + C_2 \exp(\gamma_2 x)$, as shown in the table
- $f(x) = C_1 \cosh(+\gamma_1 x) + C_2 \sinh(+\gamma_1 x)$